INVESTIGATION OF THE EFFECTS OF FALSE MATCHES AND DISTRIBUTION OF THE MATCHED KEYPOINTS ON THE PNP ALGORITHM

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ABSTRACT:

Perspective-n-Point (PnP) problem is the estimation of the pose (location and orientation) of a calibrated camera from 3D-2D point correspondences, that is, three-dimensional coordinates of objects in a world coordinate system and corresponding pixels in two-dimensional images. PnP algorithms are used in computer vision, augmented reality, robotics, photogrammetry etc. Distribution of the points in the image and the accuracy of the 3D information are important on the accuracy of the estimated pose of the camera. In this paper, keypoints with previously known coordinates and keypoints detected in images taken from a drone are matched. Then, the robustness of PnP algorithms is investigated by adding mismatched keypoints into the true matches. In addition, the effect of the distribution of the points in the image is investigated. The entire study was carried out using high-resolution orthophotos. Direct Linear Transformation (DLT), Efficient PnP (EPnP), LHM and Robust PnP (RPnP) were tested as pose estimation algorithms. As a result, it is observed that the RPnP and LHM algorithms performed better than the other pose estimation algorithms when mismatched keypoints are added. It is also observed that RPnP and LHM give accurate results when the distance error (between the true match and the false match) is increased. RPnP has an advantage over LHM in terms of computational cost. In the case of homogeneous distribution of keypoints, it is observed that PnP algorithms estimate more accurate position and orientation than in the case of nonhomogeneous distribution.

1. INTRODUCTION

PnP algorithm is the estimation of pose (position and orientation) of a calibrated camera using points from the camera images (i.e. the moving image points) and the matched 3D world coordinates (position, altitude) of these points (i.e. the fixed image (3D) points). which can be acquired from readily available maps, orthophotos, rectified aerial images etc. along with the elevation model of the acquired region (Hartley & Zisserman, 2003) “Moving points” represent keypoints detected from the image. “Fixed points” are previously detected features from reference images, so their coordinates are known. Pose estimation has a wide range of usage in robotics (Taylor & Kleeman, 2001), computer vision (Forsyth & Ponce, 2012), photogrammetry (McGlone, 2004), augmented reality etc.

This paper is dedicated to investigation of some well-known PnP algorithms by experimenting on both real and synthetic data. The entire study was carried out using high-resolution orthophotos.

The effects of number of mismatches, effects of distribution of ground control points over the moving image and effects of distance error (Figure 5) between true matches and false matches on state of art PnP algorithms were investigated in this paper.

The rest of the paper is organized as follows. The background of PnP algorithms and the necessary steps for accurate positioning before applying PnP algorithms are reviewed in Section 2 along with the details of the investigated algorithms. In Section 2.1, 2.2 and 2.3, Steps to be followed before PnP algorithms are mentioned. Due to lack of precise operation at these steps, PnP algorithm should be robust to inaccurate results of these steps. The experimental results using both synthetic data and real data are given in Section 3 and are evaluated in Section 4.

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2. BACKGROUND

Perspective n Point problem (PnP) was introduced by Fischler & Bolles (Fischler, Bolles, 1981). PnP is the estimation of camera pose using correspondence between 3D fixed points and their 2D projection. (Li, Xu, & Xie, 2012) PnP can be defined based on distances or transformations (Hu, 2002). The distance-Based Approach (Fischler et al., 1981) is the computation of the distances between control points and the optical center of the camera. Position estimation from n corresponding points and derivation of rotation and translation parameters from image plane and camera plane corresponds to Transformation-Based Approach (Horaud, Conio, Leboulleux, & Lacolle, 1989).

P3P is minimal form of PnP problem. It is solved by three points correspondence. (Grunert, 1841)

The Direct Linear Transformation was developed by (Abdel-Aziz & Karara, 1971) as a solution to the PnP problem. DLT gives accurate results with large data set.

EPnP (Lepetit, Moreno-Noguer, & Fua, 2009), one of the noniterative methods, gives accurate solution for $n \geq 3$.

LHM is one of the iterative methods, introduced by (Lu, Hager, & Mjolsness, 2000).

RPnP is introduced by Li and Chi (Li et al., 2012), which is one of the noniterative method.

In order to find the pose of a camera, the image taken from it (which will be called as “the moving image”) and a reference image (which will be called as “the fixed image”) whose world coordinates (position and altitude) are available (e.g.an orthophoto) are required. Some pose estimation algorithms (Moreno-Noguer, Lepetit, & Fua, 2008) do not need a correspondence between the moving and fixed images. This type of algorithms calculates both the pose and the correspondences simultaneously. In the scope of this study, only pose estimation algorithms are investigated. Correspondence is provided by image matching algorithms.

The most important issue for position and orientation estimation applications is robustness. Robustness refers to an ability to give correct results against drawbacks of algorithms related to image matching algorithms, outlier rejection algorithms and other related methods in the processing pipeline. One of the major drawbacks of the image matching algorithms is mismatching of detected keypoints. (Figure 4) Keypoints detected and described from two corresponding images might have similar feature vectors even if matched control points do not belong to the same location in the world. This type of matched features is named as “outliers”. PnP algorithms are influenced adversely by outliers (Ferraz, Binefa, & Moreno-Noguer, 2014). The computed location and orientation information will be inaccurate if PnP algorithm uses incorrectly matched keypoints. Therefore, outlier elimination methods are critical to get rid of incorrect matches. Distance-Based Approach is an outlier elimination method (Fischler et al., 1981). Despite using outlier rejection methods, corresponding points might include some wrong matches. Therefore, PnP algorithm also must be robust for additional wrong matches.

Furthermore, homogeneous distribution of the corresponding points in the covered image area is another crucial issue (Zhu, Wu, & Xu, 2006). In the case of homogeneous distribution of corresponding points, PnP algorithms estimate accurate position and orientation.

2.1. Internal Camera Calibration

The internal camera calibration is performed to estimate major parameters like the focal length and the parameters necessary to eliminate distortions in the image. The distortions are common problems encountered in image processing applications. Especially in vision-based navigation systems, radial distortion in the images should be corrected in order to get accurate results in the position and orientation estimation with the images taken from the cameras (Kukelova, Bujnak, & Pajdla, 2013).

The most common of these distortions are radial distortions. Radial distortions can be examined under two main headings in terms of Barrel and Pincushion Distortion.

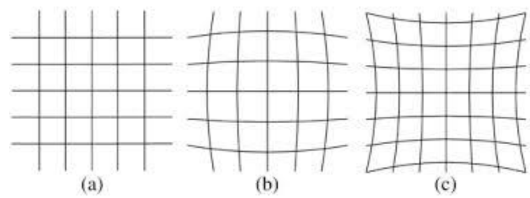
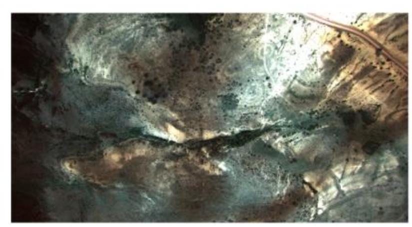
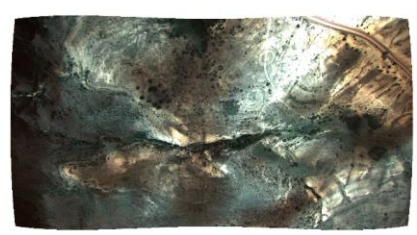


Figure 1. a) No Distortion, b) Barrel Distortion, c) Pincushion Distortion



a)



b)

Figure 2. a) Image taken by drone (distorted image) b) Undistorted Image with radial distortion coefficients

2.2. Outlier Elimination

Feature matching using descriptors computed from detected keypoints have become popular. However, local descriptor matching can produce outliers. RANSAC (Fischler et al., 1981) can robustly fit a model to data in the presence of false matches.

RANSAC is a well-known estimation method used to robustly fit a model to data in presence of outliers. (Bhattacharya & Gavrilova, 2012) Iteratively, RANSAC picks a random subset of matches from the putative match list and fits a model to them. For fitting a model, a minimum of four keypoints are required. Then, the model is compared to all other correspondences in the putative match list. (Bhattacharya & Gavrilova, 2012)

Next, keypoints which are not appropriate to the fitted model are eliminated. As we see Figure 4, features described in the moving image (left) can be matched with wrong features in the fixed image (right). After outlier rejection method, outliers can be eliminated with respect to geometric

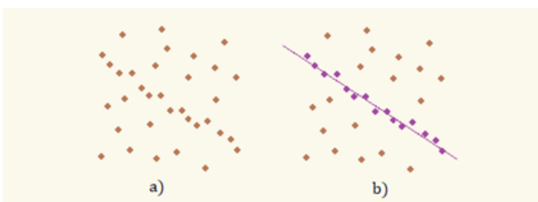
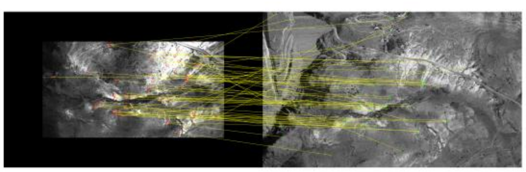
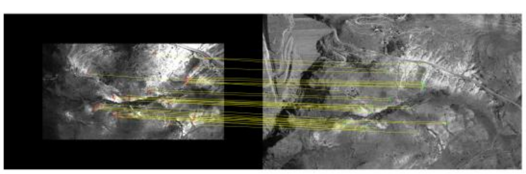


Figure 3. a) Putative data set b) Fitted line with RANSAC

transformation and fitted model by RANSAC. Despite using outlier rejection methods, corresponding points might include some wrong matches. So PnP algorithm must be robust for additional false matches.



a)



b)

Figure 4. a) Matched points before outlier rejection
b) Matched points after outlier rejection

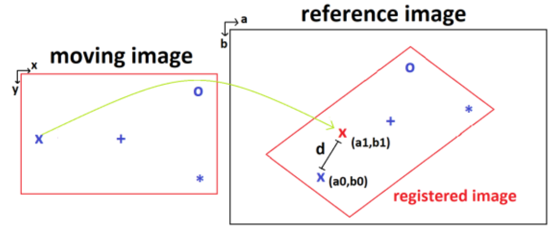


Figure 5. Distance error representation

In Figure 5, Distance error between the true match and the false match is represented as "d". (Eq 1)

$$d = \sqrt{(a_0 - a_1)^2 + (b_0 - b_1)^2} \tag{1}$$

2.3. Homogeneity

Distribution of the keypoints and accurate position estimation are directly related (Zhu et al., 2006). In the images, due to geographical characteristics of the terrain, height differences may occur from pixel to pixel. Under these circumstances, moving points should be distributed on the whole image for accurate image registration and photogrammetry.

Distribution of the keypoints is calculated using a special metric which was proposed by Yahyanejad & Rinner (Yahyanejad & Rinner, 2015). Based on this metric, in this paper, synthetic data was generated with distribution coefficient (Figure 6).

Coordinates of the keypoints are randomly generated between *(-i)* and *(i)* (Eq. 2). Totally 10 keypoints were generated.

$$P_{3D} \ : \quad \begin{aligned} x &= random[-i \ \ i]; \\ y &= random[-i \ \ i]; \\ z &= random[\,a \ \ b]; \end{aligned} \tag{2}$$

$$where, i \ \ : distribution\ coefficient$$

$$a, b \ : constant\ integer\ number \ \in Z$$

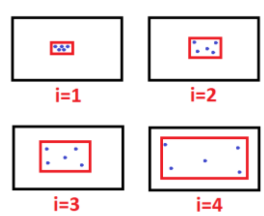


Figure 6. The changes of the distribution of the keypoints with respect to coefficient "i"

3. RESULTS

3.1. Experiments with Synthetic data

MEAN ROTATION ERROR (degree)	DLT					EPNP					LHM					RPNP				
pixel differences / Number of Outliers	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
0.5	0.0665	0.09086	0.107	0.1149	0.1151	0.0661	0.0905	0.1067	0.1144	0.1142	0.1141	0.1405	0.1088	0.1069	0.1119	0.0314	0.0427	0.054	0.0581	0.0577
20.5	2.8039	3.81936	4.339	4.9129	4.8494	2.5334	3.4101	3.7596	4.2947	4.2443	1.1314	1.5081	1.7056	1.9868	2.0011	1.3294	1.9845	2.3375	2.729	3.1925
40.5	6.0184	7.64644	9.059	10.579	10.055	5.3783	6.2371	7.4854	8.6895	8.5859	2.2279	3.0923	3.6076	3.942	3.9499	3.1175	4.6847	5.9651	6.5488	6.3724
60.5	9.1102	13.6587	14.81	16.336	15.797	7.9535	11.839	12.177	13.738	13.154	3.3257	4.6111	5.4217	5.7562	5.8832	4.7791	7.2745	8.2681	9.0972	9.2965
80.5	13.697	19.7386	22.65	23.521	24.622	11.603	16.893	19.662	20.05	20.221	4.5984	6.2464	7.8825	8.3997	8.2594	6.8052	9.8929	11.335	12.51	12.313
100.5	16.591	23.7014	29.34	30.841	31.215	14.952	21.091	23.662	24.982	23.535	5.5006	7.8396	9.5917	10.458	10.922	8.0418	12.074	14.586	15.107	15.471
120.5	21.261	29.2442	33.46	37.317	37.907	18.247	26.168	27.207	30.759	31.334	6.5684	9.4873	11.614	12.577	14.576	9.5079	13.873	16.043	17.302	18.695
140.5	24.903	35.586	41.57	42.024	43.312	23.356	31.759	36.577	38.132	34.428	8.1247	11.782	14.608	15.76	16.327	12.932	16.932	19.454	20.032	20.096
160.5	30.495	37.2746	45.53	49.177	48.322	28.356	36.497	40.48	41.781	40.102	9.6986	13.777	16.732	18.735	18.469	14.306	18	21.245	22.536	21.833
180.5	31.33	42.4817	49.48	51.883	54.821	31.356	38.831	45.415	47.233	45.436	11.211	15.451	19.346	22.783	22.234	15.72	20.266	23.195	24.448	25.36

MEAN TRANSLATION ERROR (%)	DLT					EPNP					LHM					RPNP				
pixel differences / Number of Outliers	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
0.5	0.1938	0.25134	0.279	0.297	0.3077	0.0493	0.0656	0.077	0.0831	0.0899	0.056	0.0728	0.0634	0.0634	0.0699	0.0269	0.0382	0.0502	0.0563	0.0622
20.5	7.1494	9.87951	11.4	11.604	11.99	1.7297	2.3872	2.6706	3.1926	3.2833	0.7956	1.202	1.4728	1.8258	2.0946	1.1008	1.825	2.174	2.4224	2.9738
40.5	13.597	18.2123	18.98	21.583	21.204	3.4657	4.5496	5.1042	6.2326	6.0973	1.6397	2.513	3.2154	3.9187	4.103	2.5773	3.7729	5.3244	5.7041	5.9951
60.5	19.877	27.1766	29.47	30.569	31.384	4.5435	6.9824	8.1613	9.0322	9.3545	2.5585	4.0291	5.1021	5.9342	6.5992	3.8951	6.23	7.0561	8.4889	9.0509
80.5	25.871	33.0985	36.85	35.031	38.641	6.8203	9.6792	11.538	12.69	12.893	3.645	5.9507	7.5884	8.5168	9.1918	5.5137	8.0682	9.3184	10.493	10.885
100.5	29.229	37.0258	43.24	45.363	45.322	8.6949	12.413	15.234	16.221	16.463	4.7789	7.6008	10.147	11.67	12.808	6.5859	9.9778	11.578	13.709	13.668
120.5	34.817	43.6405	48.25	49.548	50.538	11.312	15.154	18.158	19.616	20.798	5.906	10.14	12.525	14.326	16.445	7.8888	13.277	14.358	14.85	17.452
140.5	37.412	48.0308	54.14	53.232	56.163	13.835	18.959	21.964	22.749	23.307	7.2994	12.374	16.07	17.656	19.065	9.7803	15.038	17.458	17.53	19.202
160.5	42.082	54.2898	57.52	58.545	58.324	16.742	23.488	25.28	26.149	27.982	9.1777	15.147	18.44	21.232	22.538	14.716	16.249	20.1	28.086	24.298
180.5	43.794	56.1529	59.13	62.952	62.007	19.633	26.14	26.043	31.3	30.019	11.007	17.626	21	25.516	25.662	13.174	28.657	34.964	24.86	27.343

MEDIAN ROTATION ERROR (degree)	DLT					EPNP					LHM					RPNP				
pixel differences / Number of Outliers	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
0.5	0.05	0.06992	0.083	0.083	0.083	0.0497	0.0697	0.0829	0.0833	0.0825	0.0273	0.0377	0.0444	0.0443	0.0445	0.02	0.0306	0.0416	0.0423	0.0451
20.5	2.0999	2.88072	3.265	3.5252	3.5716	1.9746	2.5959	2.9001	3.1571	3.3007	0.8593	1.2375	1.3069	1.5369	1.5345	0.833	1.3288	1.5608	1.824	1.8625
40.5	4.5678	5.64513	6.714	7.0886	6.8375	3.9891	4.9137	5.7096	6.0788	5.7873	1.7505	2.4496	2.7933	3.0937	3.0146	1.6737	2.7901	3.4419	3.8367	3.9657
60.5	6.4957	9.28003	10.19	10.704	11.04	5.5685	7.4246	8.4204	9.1955	8.9089	2.6123	3.5774	4.2271	4.2544	4.5392	2.7615	4.1017	5.477	5.9533	6.044
80.5	8.6242	12.929	14.97	14.951	15.299	7.4318	10.193	11.318	11.738	12.123	3.5835	4.7363	5.7342	6.1167	6.5365	3.992	6.1443	7.7309	8.8015	8.9122
100.5	11.244	15.234	19.76	19.584	19.912	9.3536	12.971	15.312	15.371	15.176	4.2548	5.9353	7.3973	7.959	8.2648	4.7805	7.7222	10.764	10.926	11.182
120.5	13.472	19.0169	22.84	23.16	25.655	11.053	15.657	17.596	17.966	18.819	4.9794	7.2129	8.5206	8.8026	10.286	5.6597	9.0521	11.462	11.1	14.032
140.5	16.926	23.6808	28.2	28.524	29.249	13.645	18.362	22.058	21.504	21.16	6.1087	8.4796	10.703	10.78	11.189	7.5948	11.358	13.849	14.114	14.143
160.5	20.772	24.8447	31.52	34.69	34.09	16.51	22.296	23.271	25.352	23.262	7.2901	10.169	11.91	13.501	12.957	9.3928	13.155	15.522	16.898	16.845
180.5	21.891	30.0562	36.53	38.722	42.17	18.506	23.226	26.267	29.217	26.632	8.3286	11.384	13.457	15.278	15.153	11.111	14.559	17.246	18.418	19.291

MEDIAN TRANSLATION ERROR (%)	DLT					EPNP					LHM					RPNP				
pixel differences / Number of Outliers	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
0.5	0.109	0.1488	0.17	0.1909	0.1886	0.032	0.0413	0.054	0.0588	0.0657	0.0197	0.0269	0.0346	0.0426	0.0473	0.0172	0.0268	0.0369	0.0429	0.0491
20.5	4.1354	5.83645	7.284	7.7205	7.8316	1.2251	1.7293	1.9652	2.1743	2.5732	0.6114	0.9357	1.2354	1.5179	1.8404	0.67	1.0905	1.4401	1.733	2.0475
40.5	8.6788	12.26	12.72	13.845	14.011	2.2316	3.1049	3.638	4.274	4.6335	1.2489	1.9259	2.5163	3.0999	3.5397	1.4258	2.1812	2.9122	3.7368	4.0346
60.5	13.05	18.2675	20.98	21.921	22.222	3.1667	4.8175	5.6661	6.2845	7.215	1.8708	2.999	3.9366	4.8792	5.5356	2.0517	3.4127	4.6376	5.5889	6.1531
80.5	18.979	25.1081	29.68	27.166	28.858	4.2525	6.5184	7.5613	8.7065	9.4958	2.5922	4.388	5.8292	6.6981	7.7423	2.8967	4.7993	6.2781	7.7142	8.3051
100.5	23.347	30.1457	37.91	38.855	39.318	5.9362	8.4194	10.206	11.152	12.606	3.5831	5.5927	7.7672	8.6971	10.157	3.6001	5.7909	8.0695	9.4829	10.671
120.5	28.274	37.7792	45.43	45.067	46.597	6.8662	10.003	11.87	13.722	15.266	3.9645	7.7855	9.3573	11.071	12.964	4.3922	7.2748	9.4534	10.602	12.923
140.5	33.66	43.2592	53.34	51.376	54.627	8.4468	12.311	16.065	16.184	17.229	5.3139	9.2696	12.902	13.972	15.25	5.4231	8.5761	11.419	13.078	14.417
160.5	38.074	53.7827	59.74	58.422	62.144	11.133	15.633	18.16	19.081	20.655	6.6056	11.929	15.488	17.655	18.535	6.819	9.6977	11.999	14.584	16.811
180.5	40.506	58.4598	60.86	66.836	67.485	13.681	19.397	19.933	24.431	23.19	8.6154	14.526	17.208	21.167	22.383	8.2249	11.553	13.604	16.594	18.602

Table 1. Effects of Number of outliers and Distance error on PNP algorithms for synthetic data

To investigate the effects of number of false mathes on the PnP algorithms, firstly synthetic data is created. Firstly, 3D fixed points were randomly generated. Then, synthetic moving points were by projection (Eq. 3). In the experiments which were performed with synthetic data, we did not take intrinsic camera parameters, radial distortions, tangential distortions, geographical factors and other issues into account. That means pinhole camera properties are used in synthetic data and experiments. Therefore, only the effects of the false matches and the effects of distance error (Figure 5) between original points and projected points were investigated.

$$X_{2D} = \left[\frac{X_{3D}(1,:)}{X_{3D}(3,:)}; \frac{X_{3D}(2,:)}{X_{3D}(3,:)}\right] \times f \ ; \quad (3)$$

where;

$$f: focal\ length\ (in\ pixel)$$
$$X3D: randomly\ generated\ 3D\ points\ (3 \times 1\ matrix)$$
$$X2D: projected\ 2D\ moving\ points\ \ (2 \times 1\ matrix)$$

After generation of the 2D points, false matches were added to inlier points one by one. While outlier points were generated, different pixel errors were inserted. The number of outliers is 1 through 5 and, pixel error differs from 0.5 to 180.5 (Table 1).

In the first experiment (Figure 7), pixel errors were kept constant and only the number of outlier points has been increased from 1 to 5. The pixel error was fixed to 100 pixels. In total, 10 matched points were generated randomly. Number of outliers was increased one by one. Therefore, total number of matched points were constant while adding outliers.

For Figure 6	Initial Condition	Step 1	Step 2	Step 3	Step 4	Step 5
Total Number	10	10	10	10	10	10
Number of Outlier	0	1	2	3	4	5

Table 2. Steps for adding outliers to inlier points.

In the second experiment (Figure 8), the number of outliers was kept constant and only the pixel error of outliers has been increased from 0.5 to 180.5. Number of outliers was fixed to 1 in Figure 8. There were totally 10 matched points. In this situation, 9 inliers and 1 outlier were given to PnP algorithms.

For Figure 7	Initial Condition	Step 1	Step 2	Step 3	Step 4	Step 5
Total Number	10	10	10	10	10	10
Number of Outlier	0	1	1	1	1	1

Table 3. Steps for adding outliers to inlier points

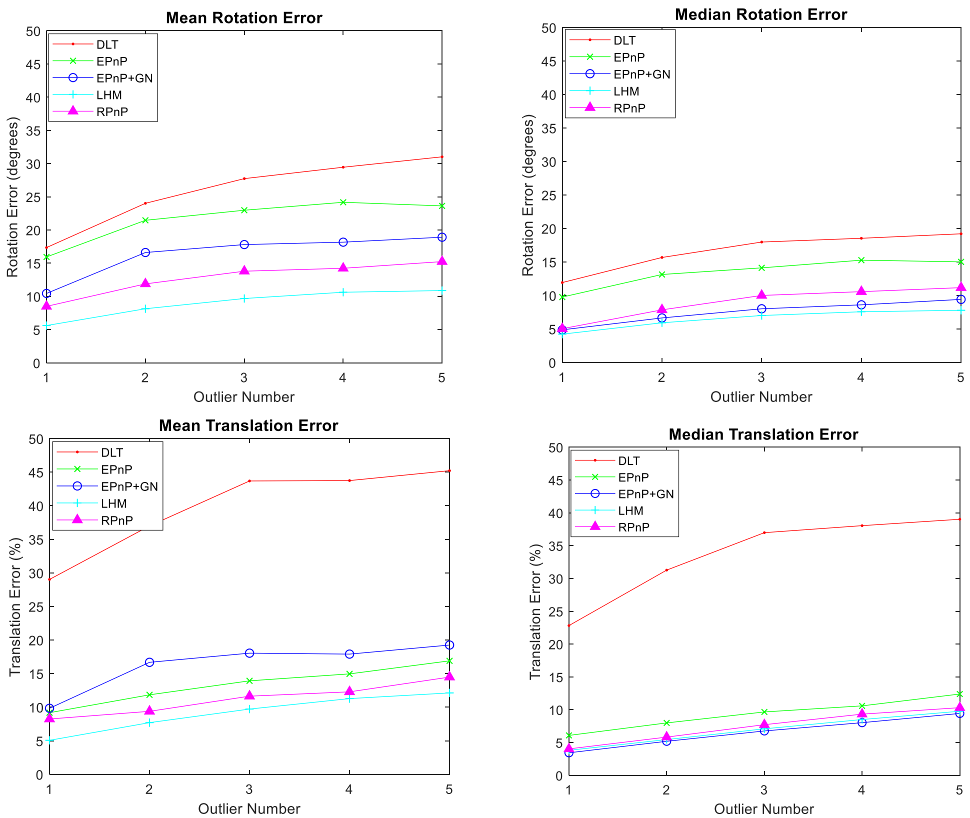


Figure 7. Effect of number of outliers on PnP algorithms

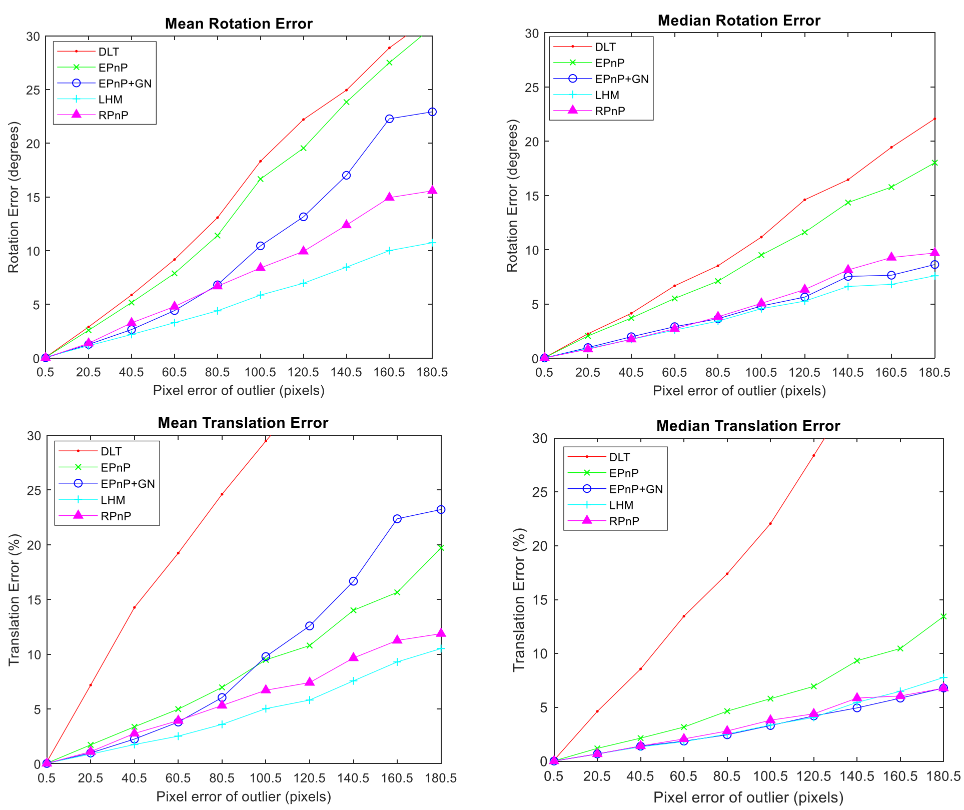


Figure 8. Effects of the pixel error on PnP algorithms

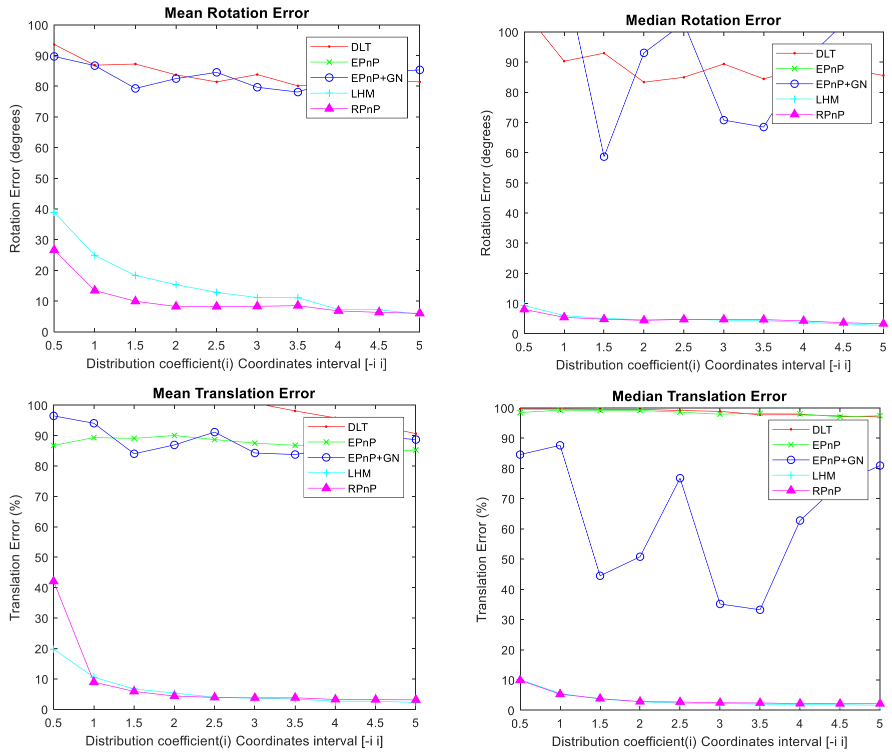


Figure 9. The effect of the distribution of the keypoints on PnP algorithms

3.2. Experiments with Real data

Distance 3D (meter)

pixel differences	DLT 1	DLT 2	DLT 3	DLT 4	DLT 5	EPNP 1	EPNP 2	EPNP 3	EPNP 4	EPNP 5	LHM 1	LHM 2	LHM 3	LHM 4	LHM 5	RPNP 1	RPNP 2	RPNP 3	RPNP 4	RPNP 5
0.5	68.191	71.017	68.51	70.715	67.017	59.539	59.565	59.592	59.516	59.549	59.385	59.389	59.401	59.398	59.418	59.252	59.261	59.242	59.283	59.215
20.5	99.813	137.1	149.99	81.186	192.41	59.298	61.128	65.262	60.928	69.1	61.081	59.866	63.262	60.902	63.235	59.629	60.611	66.624	63.571	59.286
40.5	121.48	445.29	346.77	211.54	248.93	59.51	61.425	68.576	61.21	79.661	60.663	59.617	59.667	61.376	62.516	64.63	60.938	61.207	60.72	62.496
60.5	342.45	848.91	55.973	250.7	371.85	59.748	66.398	159.7	85.635	135.22	61.698	59.594	62.158	59.909	66.997	58.958	77.331	61.664	61.758	70.934
80.5	341.32	488.62	623.92	148.02	161.62	68.318	85.757	170.1	57.255	115.34	72.376	64.226	74.782	62.659	65.36	60.493	65.097	80.847	93.614	70.337
100.5	632.21	668.25	6801.7	136.2	422.67	62.939	84.505	161.8	83.436	122.2	59.816	79.421	77.629	62.72	59.608	66.269	110.09	67.807	130.26	83.327
120.5	278.34	284.3	1402.7	1065.7	800.18	62.554	84.386	210.39	76.488	121.92	61.052	87.397	67.629	63.413	111.49	69.407	104.93	96.534	159.08	133.86
140.5	54.037	405.11	440.12	773.13	2653.7	69.95	114.69	260.36	105.62	211.99	81.2	69.475	75.826	59.88	100.16	101.79	74.202	141.56	107.65	97.796
160.5	5325.6	1458.5	927.34	2693.4	1056.3	63.789	186.63	433.57	180.99	279.96	101.72	69.468	131.78	65.839	100.79	138.18	129.95	148.98	67.146	156.34
180.5	3690.5	729.46	464.54	332.84	824.51	62.987	71.372	178.4	63.05	103.25	122.82	114.69	133.81	80.522	146.2	91.445	125.72	69.375	197.46	81.354

Distance 2D (meter)

pixel differences	DLT 1	DLT 2	DLT 3	DLT 4	DLT 5	EPNP 1	EPNP 2	EPNP 3	EPNP 4	EPNP 5	LHM 1	LHM 2	LHM 3	LHM 4	LHM 5	RPNP 1	RPNP 2	RPNP 3	RPNP 4	RPNP 5
0.5	17.913	21.203	18.084	20.956	16.612	12.638	12.609	12.772	12.607	12.706	14.051	14.039	14.019	14	14.034	14.142	14.12	14.101	14.124	14.077
20.5	51.441	137.09	149.38	42.798	191.96	13.705	20.185	20.486	15.996	32.291	15.308	16.025	24.159	18.504	22.83	10.606	17.744	29.113	24.822	16.463
40.5	80.497	379.38	328.67	171.01	245.8	17.057	22.754	41.014	16.775	50.151	14.678	15.078	15.107	19.881	25.096	34.924	13.032	18.809	18.442	22.985
60.5	332.05	803.05	34.486	200.16	306.59	17.875	33.988	140.75	57.056	110	16.281	14.462	21.128	15.645	29.36	14.2	47.717	20.483	23.81	36.01
80.5	284.56	418.78	592.98	89.896	155.9	26.684	61.609	148.68	7.7412	89.471	49.97	27.258	43.934	22.274	31.263	15.191	28.08	51.926	67.379	35.927
100.5	593.69	581.87	6168.1	99.351	419.91	23.183	60.029	144.49	53.572	96.902	24.699	50.507	48.375	22.014	11.885	25.906	84.201	25.665	106.35	55.139
120.5	225.78	277.02	1369.9	1045.4	768.74	13.362	39.691	197.48	48.04	100.81	22.213	61.508	31.888	20.771	84.466	30.68	77.106	71.119	135.23	113.53
140.5	16.837	306.37	425.29	739.54	2419.3	22.459	97.676	228.52	79.471	182.8	44.332	37.324	46.476	14.333	78.585	67.576	40.408	110.39	81.212	70.617
160.5	4899.2	1408.4	639.01	2539.1	952.14	29.502	124.76	326.05	171.74	249.67	88.451	35.461	108.64	27.783	77.907	125.98	108.78	125.26	30.355	134.57
180.5	3439	623.12	368.4	332.79	211.98	8.4112	41.021	165.71	33.908	71.183	89.018	89.183	108.75	50.88	126.37	72.628	97.396	33.314	167.91	41.743

Table 4. Effects of Number of outliers vs Distance error on PnP algorithms for real data

To investigate effects of number of false mathes on the PnP algorithms using real data, corresponding ground control points are picked manually as keypoints. For every image pair, 10 handpicked points were chosen. Real data consist of 2D and 3D data set which are selected from images taken from an aerial vehicle and orthophotos available for the flight region. In the experiments with real data, intrinsic camera parameters, radial distortions, tangential distortions, geographical factors, and other issues were taken into account. To remove radial distortion and tangential distortion, moving points were undistorted by *MATLAB Camera Calibration Toolbox*. After undistorting the 2D points, false matches were added to inlier points one by one. While outlier points were generated, different pixel errors were inserted. The number of outliers is 1 through 5 and, pixel error differs from 0.5 to 180.5 (Table 4). In Figure 11, Pixel errors were kept constant and only the number of outlier points has been increased from 1 to 5. The pixel error was fixed to 100 pixels in Figure 11. Number of outlier was increased one by one. Therefore total number of matched points were constant while adding outlier. In Figure 10, number of outlier was kept constant and only the distance error between true match and outlier has been increased from 0.5 to 500.

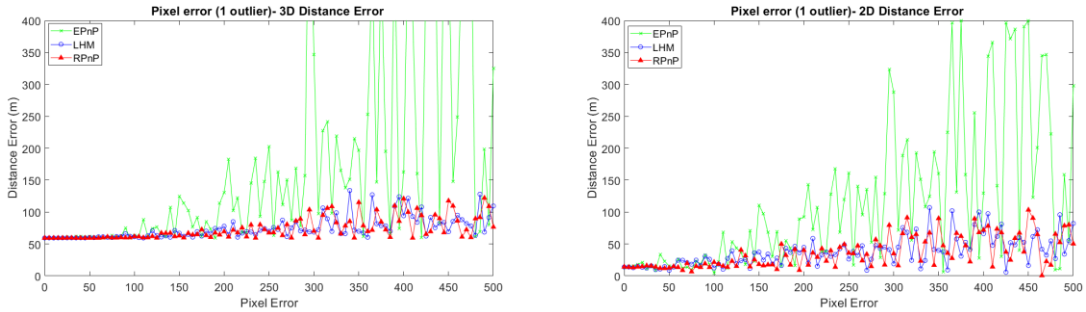


Figure 10. Effects of the pixel error on PnP algorithms

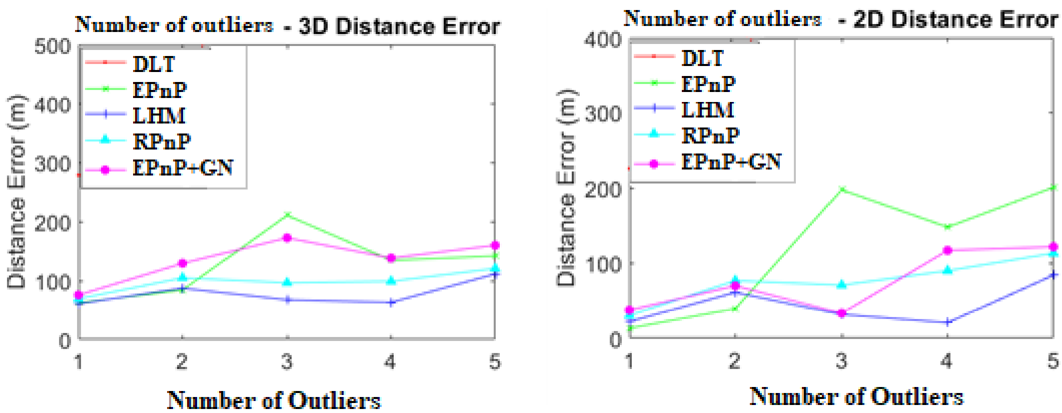


Figure 11. Effect of number of outliers on PnP algorithms

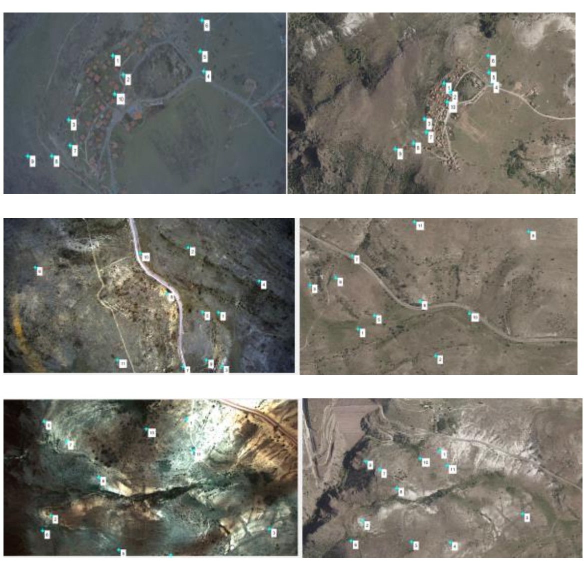


Figure 12. Some examples of the Image data set along with the handpicked keypoints

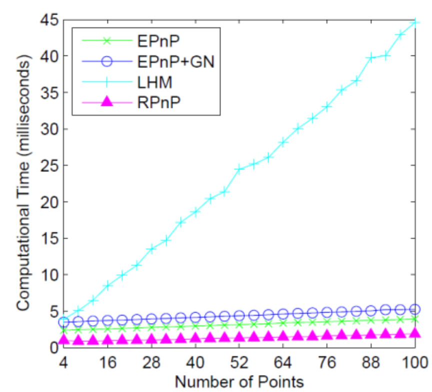


Figure 13. Computational cost comparison (Li et al., 2012)

According to synthetic experimental results, LHM and RPnP are robust methods to increasing number of outliers. The performance of PnP algorithms is affected adversely by increasing distance error (between true match and outlier) (Figure 5). RPnP and LHM also are robust to distance error. In the case of homogeneous distribution, the performance of all PnP algorithms is enhanced. Results obtained from synthetic experiments are satisfied by experiments on real data.

4. CONCLUSION

In this paper, well-known PnP algorithms were investigated and compared with synthetic and real data in terms of effects of false matches, effect of distribution of the matched keypoints, and the effects of distance error between original points and projected points. Experiments with synthetic and real data show that RPnP and LHM can effectively cope with data sets which include drawbacks investigated in this paper. Although LHM generally works better than RPnP under some drawbacks, the computational time is considerably longer for use in real-time applications. (Figure 13) Therefore, it can cause problems in real-time applications. So RPnP is a very efficient and robust method for photogrammetry and computer vision applications especially real-time applications.

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REFERENCES

Abdel-Aziz, Y. I., & Karara, H. M. (1971). Direct Linear Transformation from Comparator Coordinates into Object Space Coordinates in Close-Range Photogrammetry. *Photogrammetric Engineering & Remote Sensing*. https://doi.org/10.14358/PERS.81.2.103

Bhattacharya, P., & Gavrilova, M. (2012). Improving RANSAC feature matching with local topological information. *Proceedings of the 2012 9th International Symposium on Voronoi Diagrams in Science and Engineering, ISVD 2012*. https://doi.org/10.1109/ISVD.2012.8

Ferraz, L., Binefa, X., & Moreno-Noguer, F. (2014). Very fast solution to the PnP problem with algebraic outlier rejection. *Proceedings of the IEEE Computer Society Conference on Computer Vision and Pattern Recognition*. https://doi.org/10.1109/CVPR.2014.71

Fischler, M. A., Fischler, M. A., Bolles, R. C., & Bolles, R. C. (1981). Random sample consensus. A paradigm for model fitting with apphcahons to image analysm and automated cartography. *Graphics And\nImage Processing*.

Forsyth, D. A., & Ponce, J. (2012). COMPUTER VISION A MODERN APPROACH. In *Uma ética para quantos?* https://doi.org/10.1007/s13398-014-0173-7.2

Hartley, R., & Zisserman, A. (2003). Multiple View Geometry in Computer Vision in computer vision. *Computer-Aided Design*.

Horaud, R., Conio, B., Leboulleux, O., & Lacolle, B. (1989). An analytic solution for the perspective 4-point problem. *Computer Vision, Graphics and Image Processing*. https://doi.org/10.1016/0734-189X(89)90052-2

Hu, Z. (2002). A note on the number of solutions of the noncoplanar P4P problem. *IEEE Transactions on Pattern Analysis and Machine Intelligence*

Kukelova, Z., Bujnak, M., & Pajdla, T. (2013). Real-time solution to the absolute pose problem with unknown radial distortion and focal length. *Proceedings of the IEEE International Conference on Computer Vision*. https://doi.org/10.1109/ICCV.2013.350

Lepetit, V., Moreno-Noguer, F., & Fua, P. (2009). EPnP: An accurate O(n) solution to the PnP problemLepetit, V.,

Moreno-Noguer, F., & Fua, P. (2009). EPnP: An accurate O(n) solution to the PnP problem. International Journal of Computer Vision. https://doi.org/10.1007/s11263-008-0152-6. *International Journal of Computer Vision*. https://doi.org/10.1007/s11263-008-0152-6

Li, S., Xu, C., & Xie, M. (2012). A robust O(n) solution to the perspective-n-point problem. *IEEE Transactions on Pattern Analysis and Machine Intelligence*. https://doi.org/10.1109/TPAMI.2012.41

Lu, C. P., Hager, G. D., & Mjolsness, E. (2000). Fast and globally convergent pose estimation from video images. *IEEE Transactions on Pattern Analysis and Machine Intelligence*. https://doi.org/10.1109/34.862199

McGlone, C. M. (2004). Manual of photogrametry. *In American society for photogrammetry and remote sensing.*

Moreno-Noguer, F., Lepetit, V., & Fua, P. (2008). Pose priors for simultaneously solving alignment and correspondence. *Lecture Notes in Computer Science (Including Subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics)*. https://doi.org/10.1007/978-3-540-88688-4-30

Taylor, G., & Kleeman, L. (2001). Flexible self-calibrated visual servoing for a humanoid robot. *Proceedings of the Australian Conference on Robotics and Automation*.

Yahyanejad, S., & Rinner, B. (2015). A fast and mobile system for registration of low-altitude visual and thermal aerial images using multiple small-scale UAVs. *ISPRS Journal of Photogrammetry and Remote Sensing*. https://doi.org/10.1016/j.isprsjprs.2014.07.015

Zhu, Q., Wu, B., & Xu, Z. X. (2006). Seed point selection method for triangle constrained image matching propagation. *IEEE Geoscience and Remote Sensing Letters*. https://doi.org/10.1109/LGRS.2005.861735

Grunert, J. A. (1841). Das pothenotische problem in erweiterter gestalt nebst bber seine anwendungen in der geodasie. *Grunerts Archiv fur Mathematik und Physik*, 238-248.